TRANSPORT OF RADIANT ENERGY IN MULTILAYER SCATTERING AND ABSORBING MATERIALS

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UDC 536.3

By taking account of boundary reflection we solve the problem of the distribution of absorbed energy and radiation fluxes over the thickness of individual layers in realistic multilayer absorbing and scattering materials.

Investigations of radiative heat-transfer processes [1] and the transport of energy in multilayer systems [3, 4, 7, 8, 10-14, 16, 17] have become very important, since actual objects exposed to infrared radiation are systems of several layers of selectively absorbing and scattering materials (multilayer protective shells and coverings for meteorological and cosmic devices and equipment, building materials, outer shells of vegetable materials, biological objects, food products, etc.). Efficient methods of calculating heat transfer [9] in infrared radiation processes leading to the formation of multilayer systems in the presence of moving phase transition boundaries are also important for practical application in various fields of science and technology. Therefore it is important to know the laws for the attenuation of radiation fluxes in multilayer systems, taking account of boundary reflection and selective absorption and scattering of radiation from optical inhomogeneities which appreciably change the distribution of radiant energy within individual layers [7, 15] and the values of the integrated thermoradiative characteristics of each layer and of the whole system.

Special problems are discussed in [4, 8, 11, 14, 16, 17]: a stack of nonscattering layers [8, 16], a weakly scattering thin film between metal layers [17], systems of scattering layers [4, 14], and a two-layer purely scattering spherical atmosphere [11] neglecting the effects of boundary reflection.

It is expedient to solve the general problem of the transport of radiant energy in a multilayer system by a refined differential-difference method [7], taking account of the effects of multiple scattering and boundary reflection, the irradiation conditions, and the selectivity of the optical and emission properties of all bodies involved in the heat transfer. Using this method Il'yasov and Krasnikov [7] obtained the laws of attenuation of the radiation flux, taking account of multiple scattering and boundary reflection in a plane layer on an opaque substrate.

We consider the general case of the irradiation of a system of layers from both sides by diffuse monochromatic radiation having flux densities E_{I} and E_{II} (Fig. 1). We propose the following methods for finding the counterflow radiation flux densities E_i^+ and E_i^- within the i-th layer of a multiplayer system: the method of combining absorbing and scattering layers by taking account of boundary reflections, and the method of combining layers by representing the boundaries as fictitious nonabsorbing layers having asymmetric reflection coefficients $\rho_{i,k} \neq \rho_{k,i}$ and transmission coefficients

$$t_{i,k} = (1-\rho_{i,k}) \neq t_{k,i} = (1-\rho_{k,i}). \tag{1}$$

In the method of combining layers by taking account of boundary reflection in the boundary conditions each layer i can be considered as being irradiated from both sides by the external fluxes $E_{i,\mathrm{I}}$ and $E_{i,\mathrm{II}}$, related to the counterradiation fluxes E_i^+ and E_i^- by the following boundary conditions (Fig. 1):

$$\text{for}\quad x_i = 0 \quad E_{i,\mathrm{I}} = E_{i-1}^+(l_{i-1}),$$
$$E_i^+(0) = (1-\rho_{i,i-1})\,E_{i-1}^+(l_{i-1}) + \rho_{i-1,i}E_i^-(0), \tag{2}$$

Moscow Technological Institute of the Food Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 6, pp. 1053-1060, December, 1974. Original article submitted July 10, 1974.

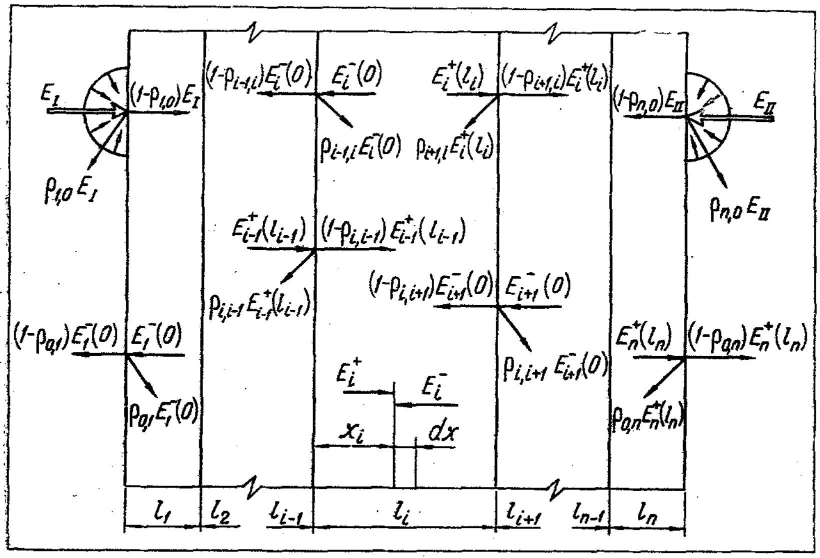

Fig. 1. Radiation fluxes in a multilayer system of selectively absorbing and scattering materials.

$$\text{for } x_i = l_i \quad E_{i,\mathrm{II}} = E_{i+1}^-(x_{i+1} = 0),$$

$$E_i^-(l_i) = \rho_{i+1,i}E_i^+(l_i) + (1 - \rho_{i,i+1})E_{i+1}^-(0). \tag{3}$$

The coefficients of boundary reflection $\rho_{i,k}$ in (2) and (3) depend on the index of refraction n_λ and the absorption coefficient k_λ, and are determined by the Fresnel formulas. The first subscript denotes the number of the irradiated layer and the second the number of the layer from which the flux is incident. To calculate the quantities $\rho_{i,k}$ in the present case of irradiation of the system from both sides by diffuse radiation fluxes the angular distribution of the fluxes incident on the boundaries of the i-th layer within the system can also be assumed diffuse as a consequence of the effects of multiple reflections between layers and multiple scattering within the layers.

The optical properties of the material of the i-th layer are characterized by the spectral absorption coefficient $\bar{k}_i$ and the backscattering coefficient s_i averaged over a half-space. These coefficients give complete information on the spatial distribution of radiant energy within the layer, the optical properties of the material, and are related to the basic absorption and scattering characteristics $k_{\lambda,i}$ and $\sigma_{\lambda,i}$ by the equations

$$\bar{k}_i = m_i k_{\lambda,i}, \quad s_i = m_i \delta_{s,i}\sigma_{\lambda,i}. \tag{4}$$

The auxiliary coefficients δ_s and m take account of the irradiation conditions for an elementary layer of thickness dx and the form of the scattering indicatrix ($m \leqslant 2$, $\delta_s \leq 1$ [7]).

In this case for any layer i of thickness l_i ($0 \leq x_i \leq l_i$) of a multilayer system (Fig. 1) we obtain a familiar system of linear differential equations for the hemispherical counterfluxes E_i^+ and E_i^- (system (36) on p. 31 of [7]).

The general solution of the system of differential equations (36) of [7] for the counterfluxes in the i-th layer for boundary conditions (2) and (3) have the form

$$E_i^+(x_i) = \frac{E_{i,\mathrm{I}}C_{i,i-1}}{1 - B_{i-1,i}B_{i+1,i}\Psi_i^2}[\exp(-L_i x_i) - B_{i+1,i}\Psi_i^2\exp(L_i x_i)]$$

$$+ \frac{E_{i,\mathrm{II}}C_{i,i+1}\Psi_i}{1 - B_{i-1,i}B_{i+1,i}\Psi_i^2}[\exp(L_i x_i) - B_{i-1,i}\exp(-L_i x_i)], \tag{5}$$

$$E_i^-(x_i) = \frac{E_{i,\mathrm{I}}C_{i,i-1}\Psi_i}{1 - B_{i-1,i}B_{i+1,i}\Psi_i^2}\{\exp[L_i(l_i - x_i)] - B_{i+1,i}\exp[-L_i(l_i - x_i)]\}$$

$$+ \frac{E_{i,\mathrm{II}}C_{i,i+1}}{1 - B_{i-1,i}B_{i+1,i}\Psi_i^2}\{\exp[-L_i(l_i - x_i)] - B_{i-1,i}\Psi_i^2\exp[L_i(l_i - x_i)]\}. \tag{6}$$

The symbols L and ψ in (5) and (6) are in accord with [7]. The coefficients $C_{i,k}$ and $B_{i,k}$, taking account of reflection at the boundaries of the i-th layer, are given by the equations

$$C_{i,i\pm 1} = \frac{1-\rho_{i,i\pm 1}}{1-\rho_{i\pm 1,i} R_{i\infty}}, \tag{7}$$

$$B_{i\pm 1,i} = \frac{R_{i\infty} - \rho_{i\pm 1,i}}{R_{i\infty}(1-\rho_{i\pm 1,i} R_{i\infty})}. \tag{8}$$

The flux densities of the radiations $E_{i,\mathrm{I}}$ and $E_{i,\mathrm{II}}$ incident on the boundaries of the i-th layer ($1 < i < n$), taking account of multiple reflections, are expressed in terms of the densities of the external fluxes E_I and E_II incident on the system by the equations

$$E_{i,\mathrm{I}} = [E_\mathrm{I}\, T_{1\ldots(i-1),0} + E_\mathrm{II} T_{n\ldots i,0} R_{(i-1)\ldots 1,i}] \frac{1}{1-R_{(i-1)\ldots 1,i} R_{i\ldots n,(i-1)}}, \tag{9}$$

$$E_{i,\mathrm{II}} = [E_\mathrm{II} T_{n\ldots(i+1),0} + E_\mathrm{I}\, T_{1\ldots i,0} R_{(i+1)\ldots n,i}] \frac{1}{1-R_{(i+1)\ldots n,i} R_{i\ldots 1,(i+1)}}. \tag{10}$$

The densities of the fluxes incident on the first and n-th layers, taking account of conditions (9) and (10), are: $E_{i,\mathrm{I}} = E_\mathrm{I}$ and $E_{n,\mathrm{II}} = E_\mathrm{II}$

$$E_{1,\mathrm{II}} = [E_\mathrm{II} T_{n\ldots 2,0} + E_\mathrm{I}\, T_{1,0} R_{2\ldots n,1}] \frac{1}{1-R_{2\ldots n,1} R_{1,2}}, \tag{11}$$

$$E_{n,\mathrm{I}} = [E_\mathrm{I}\, T_{1\ldots(n-1),0} + E_\mathrm{II} T_{n,0} R_{(n-1)\ldots 1,n}] \frac{1}{1-R_{(n-1)\ldots 1,n} R_{n,(n-1)}}. \tag{12}$$

Equations (5) and (6) give the fluxes of monochromatic radiation at depth x_i in a multilayer system, taking account of the effects of external and internal reflection from the boundaries of the i-th layer, and have a more general form than those obtained earlier for a single layer in air [7]. They make it possible to obtain general formulas for the thermoradiative characteristics of a layer bounded by different layers (media) within a multilayer system.

The transmittance and reflectance of the i-th layer are determined from (5) and (6) for the condition $E_{i,\mathrm{II}} = 0$:

$$T_i = (1-\rho_{i+1,i}) \frac{E_i^+(l_i)}{E_{i,\mathrm{I}}} (E_{i,\mathrm{II}} = 0) = (1-\rho_{i+1,i}) \frac{C_{i,i-1}(1-B_{i+1,i} R_{i\infty}^2)}{1-B_{i-1,i} B_{i+1,i} \Psi_i^2} \exp(-L_i l_i), \tag{13}$$

$$R_i = \rho_{i,i-1} + (1-\rho_{i-1,i}) \frac{E_i^-(0)}{E_{i,\mathrm{I}}} (E_{i,\mathrm{II}} = 0) = \rho_{i,i-1} + (1-\rho_{i-1,i})\, C_{i,i-1} R_{i\infty} \frac{1-B_{i+1,i}\exp(-2L_i l_i)}{1-B_{i-1,i} B_{i+1,i} \Psi_i^2}. \tag{14}$$

In the special case of a single layer in air or vacuum $\rho_{i,i-1} = \rho_{1,0}$, $\rho_{i+1,i} = \rho_{0,1}$ and Eqs. (13) and (14) agree with the well-known equations of [5, 7].

The amount of radiant energy absorbed per unit time at depth x_i by an elementary volume of thickness dx is determined from the equation of conservation of energy [1, 7]:

$$w_i(x_i) = \bar{k}_i [E_i^+(x_i) + E_i^-(x_i)] = \bar{k}_i E_{i,0}(x_i), \tag{15}$$

where $E_{i,0}(x_i) = E_i^+ + E_i^-$ is the spatial irradiance at depth x_i

$$w_i(x_i) = \bar{k}_i E_{i,\mathrm{I}} \frac{(1+R_{i\infty}) C_{i,i-1}}{1-B_{i-1,i} B_{i+1,i} \Psi_i^2} \left[\exp(-L_i x_i) - B_{i+1,i} \frac{\Psi_i^2}{R_{i\infty}} \exp(L_i x_i)\right]$$
$$+ \bar{k}_i E_{i,\mathrm{II}} \frac{(1+R_{i\infty}) C_{i,i+1}}{1-B_{i-1,i} B_{i+1,i} \Psi_i^2} \left\{\exp[-L_i(l_i - x_i)] - B_{i-1,i} \frac{\Psi_i^2}{R_{i\infty}} \exp[L_i(l_i - x_i)]\right\}. \tag{16}$$

Thus, the problem of determining the radiation field in a multilayer system is reduced to the problem of finding the transmittances and reflectances of the system, taking account of boundary reflection, by the method of combining layers [5-8, 14, 16]: $T_{1\ldots(i-1),0}$, $T_{1\ldots i,0}$ are the transmittances of the system of layers from the first to the (i − 1)-th and to the i-th for external radiation to the first layer; $T_{n\ldots i,0}$, $T_{n\ldots(i+1),0}$ are the transmittances of the system of layers from the n-th to the i-th and to the (i + 1)-th for external radiation to the n-th layer: $R_{(i-1)\ldots 1,i}$, $R_{i\ldots 1,i}$, $R_{i\ldots n,i}$, $R_{(i+1)\ldots n,i}$ are the reflectances of the stack of layers for internal irradiation of the corresponding boundary of the i-th layer.

The thermoradiative characteristics of a multilayer system, taking account of absorption, multiple scattering, and boundary reflection, are determined from the general solution of (5) and (6) by using (11) and (12). Under the condition $E_{II} = 0$ we find from (5) by using (11) the following formula for the reflectance of a multilayer system:

$$R_{1\ldots n} = \rho_{1,0} + (1-\rho_{0,1})\frac{E_I^-(0)}{E_I}(E_{II}=0) = \rho_{1,0} + R_{1,0} + \frac{R_{2\ldots n,1}T_{1,0}T_{1,2}}{1-R_{2\ldots n,1}R_{1,2}}. \tag{17}$$

The transmittance of a multilayer system is found from (6) by using (12) and the condition $E_{II} = 0$:

$$T_{1\ldots n} = (1-\rho_{0,n})\frac{E_n^+(l_n)}{E_I}(E_{II}=0) = \frac{T_{1\ldots(n-1),0}T_{n,n-1}}{1-R_{(n-1)\ldots 1,n}R_{n,(n-1)}}. \tag{18}$$

The absorptance of a multilayer system is determined from the expression

$$A_{1\ldots n} = 1-(T_{1\ldots n} + R_{1\ldots n}). \tag{19}$$

Equations (17) and (18) have the form of the equations for a pile of nonscattering plates [16] and for a system of scattering layers, neglecting boundary reflection [14].

The effect of boundary reflection on the propagation of radiant energy in a multilayer system and on the values of its thermoradiative characteristics can be established by the method of combining layers which takes account of boundary reflection by representing boundaries as fictitious nonabsorbing layers having asymmetric reflection coefficients $\rho_{i,k} \neq \rho_{k,i}$ and transmission coefficients $t_{i,k} \neq t_{k,i}$. According to the method proposed an isolated i-th layer is represented as a set of three layers: two boundary layers i, i − 1 and i, i + 1, and the i-th layer itself having a reflectance R_i and a transmittance T_i determined from known expressions [5, 7] which neglect boundary reflection,

$$T_i = \frac{1-R_{i,\infty}^2}{1-\Psi_i^2}\exp(-L_i l_i), \tag{20}$$

$$R_i = R_{i\infty}\frac{1-\exp(-2L_i l_i)}{1-\Psi_i^2}. \tag{21}$$

In the present case it can be assumed that on an isolated i-th layer inside the system there are incident from both sides fluxes with densities $E_{i,1} = E_i^+(0)$ and $E_{i,2} = E_i^-(l_i)$ related to the external flux densities $E_{i,I} = E_{i-1}^+(l_{i-1})$ and $E_{i,II} = E_{i+1}(x_{i+1} = 0)$ incident on the boundaries of this layer by the conditions

$$E_{i,1}(x_i=0) = E_i^+(0) = (1-\rho_{i,i-1})E_{i,I} + \rho_{i-1,i}E_i^-(0), \tag{22}$$

$$E_{i,2}(x_i=l_i) = E_i^-(l_i) = (1-\rho_{i,i+1})E_{i,II} + \rho_{i+1,i}E_i^+(l_i). \tag{23}$$

The solution of Eqs. (36) of [7] for the counterfluxes for boundary conditions (22) and (23) are Eqs. (38) and (39) of [7]. There must be substituted into these expressions the following values of the flux densities $E_{i,1}$ and $E_{i,2}$ found from (22) and (23) by using (38) and (39) of [7]. These establish the effect of boundary reflection on the distribution of radiation fluxes within the system of layers:

$$E_{i,1} = \frac{C'_{i,i-1}}{1-D_{i+1,i}D_{i-1,i}}E_{i,I} + \frac{C'_{i,i+1}D_{i-1,i}}{1-D_{i+1,i}D_{i-1,i}}E_{i,II}, \tag{24}$$

$$E_{i,2} = \frac{C'_{i,i+1}}{1-D_{i+1,i}D_{i-1,i}}E_{i,II} + \frac{C'_{i,i-1}D_{i+1,i}}{1-D_{i+1,i}D_{i-1,i}}E_{i,I}, \tag{25}$$

where

$$C'_{i,k} = \frac{1-\rho_{i,k}}{1-\rho_{k,i}R_i}; \quad D_{k,i} = \frac{\rho_{k,i}T_i}{1-\rho_{k,i}R_i}. \tag{26}$$

The function $w_i(x)$ for the i-th layer of the system is determined from (15) by using solutions (38) and (39) of [7]:

$$w_i(x_i) = \bar{k}_i E_{i,1} \frac{1 - R_{i\infty}}{1 - \Psi_i^2} \left[\exp(-L_i x_i) - \frac{\Psi_i^2}{R_{i\infty}} \exp(L_i x_i) \right] + \bar{k}_i E_{i,2}$$
$$\times \frac{1 - R_{i\infty}}{1 - \Psi_i^2} \left\{ \exp[-L_i(l_i - x_i)] - \frac{\Psi_i^2}{R_{i\infty}} \exp[L_i(l_i - x_i)] \right\}. \quad (27)$$

A comparison of functions (27) and (16) shows that the method of representing boundaries as reflecting and transmitting layers gives a simpler intuitive solution. Here the total number of layers increases and becomes equal to $2n + 1$, which complicates the calculation of $E_{i,\mathrm{I}}$ and $E_{i,\mathrm{II}}$ in a multilayer system. In addition, the method makes it possible to establish a relation between the thermoradiative characteristics R_i and T_i of the i-th layer calculated by taking account and not taking account of boundary reflection.

By using Eqs. (24), (25) and (20), (21) with the known solution (38) and (39) of [7], general expressions can be written for the reflectance and transmittance of the i-th layer within the system, taking account of boundary reflections:

$$T_i(\rho_{i,k}, \rho_{k,i}) = \frac{C'_{i,i-1} T_i}{1 - D_{i+1,i} D_{i-1,i}} \left(1 + \frac{D_{i+1,i} R_i}{T_i} \right), \quad (28)$$

$$R_i(\rho_{i,k}, \rho_{k,i}) = \frac{C'_{i,i-1} R_i}{1 - D_{i+1,i} D_{i-1,i}} \left(1 + \frac{D_{i+1,i} T_i}{R_i} \right). \quad (29)$$

The effect of boundary reflection on the absorptance can be established by comparing the equations

$$A_i = 1 - (R_i + T_i), \quad (30)$$

$$A_i(\rho_{i,0}, \rho_{0,i}) = 1 - (R_i + T_i) \frac{(1 - \rho_{i,0}) \rho_{0,i} T_i}{(1 - \rho_{0,i} R_i)^2 - \rho_{0,i}^2 T_i^2}. \quad (31)$$

The values of the reflection coefficients at the boundaries $\rho_{i,k}$ for an external flux incident on the i-th layer, and $\rho_{k,i}$ for the flux emerging from the layer, are different.

For a strongly scattering medium with $\Lambda_e = 0.9$, $\varepsilon l = 1.0$, $\rho_{i,0} \simeq 0.1$, $\rho_{0,i} \simeq 0.4$ [5, 7] we find that the absorptance $A_i(\rho_{i,0}, \rho_{0,i}) = 0.75$ calculated by (31) taking account of boundary reflection is 8.35 times as large as the value $A_i = 0.09$ calculated by (30) without taking account of boundary reflection (an error of ~800%). With an increase in n_λ and Λ_e the values of $\rho_{0,i}$ and R_i increase, which leads to a larger error in determining the absorptance of a multilayer system of materials without taking account of boundary reflection. Since for many materials n_λ lies in the 1.1-5.0 range [7, 12] and Λ_e reaches 0.99 [7], ignoring reflection at the boundary can lead to results which do not correspond to the actual heat-transfer process. In this connection, in determining the incident flux density E_i, the resultant E_r, and the effective E_{eff} at the surface of a multilayer system or on the boundary of the i-th layer within the system it is necessary to substitute into the known equations [1] the value of A_s of the system found by the proposed method, taking account of boundary reflection.

A comparison of the general solutions (5), (6), (16), (27) and (38), (39) of [7] shows that internal boundary reflection makes appreciable changes in both the distribution of absorbed energy and the radiation fluxes over the thickness of the i-th layer within a multilayer system. This is taken into account in the solutions by the coefficients $C_{i,k}$, $B_{i,k}$, $C'_{i,k}$, and $D_{i,k}$ which depend on $\rho_{i,k}$, $R_{i,\infty}$, R_i, and T_i.

NOTATION

R, T, A	are the reflectance, transmittance, and absorptance of a layer of thickness l;
L, $\bar{k}$, s	are the average coefficients of effective attenuation, absorption, and backscattering of an elementary layer of thickness dx;
$\rho_{i,k}$	is the coefficient of boundary reflection;
E, E^+, E^-	are the densities of the incident and counterradiation fluxes, W/m^2;
E_0	is the spatial irradiance within a layer, W/m^3;
$\Lambda_e = S/(k + S)$;	
i, n, k	are the layer-number subscripts;
1. . . n	is the system of layers;
λ	is the spectral;
0	is the air.

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